

# A Logic-Based Approach for Real-Time Object-Oriented Software Development

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## Abstract

This paper discusses how RETOOL, an action logic featuring an operator that expresses necessary conditions, postconditions and time bounds of actions, can be combined with MTL, a linear-time temporal logic with time-bounded operators, to reason about general properties of timed transition systems, an abstract model for the behavior of objects in a real-time, object-oriented software system.

**Keywords:** Modal Logic in Computing; Action Logic; Timed Transition Systems; Metric Temporal Logic

## 1 Introduction

This paper is inserted in the context of an environment for real-time object-oriented software development based on Formal Methods. The framework depicted here has been considered for use at a research laboratory at PUC-RJ in the development of the prototype of a CASE environment for an industrial partner in the field of telecommunications ([9]).

The basic architecture of the environment consists of a set of tools implemented over an integration platform (currently CORBA).

Conceptually, the environment consists of several planes that relate the tools to various aspects of functionality. It has a user plane, a formal plane and an implementation plane. In the user plane, software designers model applications and libraries via class relationships, inter-object communication and state diagrams. The user plane can be used to produce a so-called *scenario* (i.e., fixing parameters in the design to specific values), which is then used by a model checker (SMV – Symbolic Model Verifier [6]) for verifying properties of the design. The formal plane is also a target for a mapping from designs to the action logic RETOOL ([1, 4, 7]) in order to support general reasoning about the design, as opposed to verification and validation tasks. The implementation plane is responsible for deriving code, through transformation rules, in a high level programming language (DDL – see [3]) associated with the diagrammatic class notation of the environment. Finally, executable code is generated from the DDL code, again through transformation rules.

The focus of this paper is on the support that the formal plane was designed to provide for object-oriented development. More specifically, the paper concentrates on the way the

logic RETOOL allows reasoning about the enabling conditions, postconditions and time bounds of the actions involved in the design. The version of RETOOL presented here is sound and complete, as opposed to the tentative axiomatizations of [4] and [7].

Section 2 discusses the action logic RETOOL and its combination with a linear-time temporal logic (MTL – see [5]) to abstract temporal properties of actions; section 3 introduces the concrete model of Timed Action Transition Diagrams, used to represent designs; finally, section 4 provides an example of the use of these formalisms to reason about a specific design.

## 2 The Logics

This section presents the action logic RETOOL in detail and discusses how it can be combined with a temporal logic (MTL) to abstract temporal properties of actions.<sup>1</sup>

### 2.1 RETOOL: the Language

The primitive syntactic entities of RETOOL are *attribute symbols* (which, in this propositional version of the logic, are simple propositional letters), and *action symbols*. We denote the set of attribute symbols as  $A$ , and the set of action symbols as  $\Gamma$ .

The logic presupposes an infinite totally ordered set  $(\text{TIME}, \leq)$ , with minimum 0. A constant  $\infty$  is available, such that  $\infty \notin \text{TIME}$  and  $t \leq \infty, \forall t \in \text{TIME}$ .<sup>2</sup> The other syntactic categories are:

- State Propositions ( $SP$ ):  $p ::= a \mid \neg p \mid p \rightarrow p'$ , where  $a \in A$ ;
- Action terms ( $AT$ ):  $t ::= g \mid p_l \delta^u q$ , where  $g \in \Gamma$ ,  $p, q \in SP$ ,  $l \in \text{TIME}$ ,  $u \in \text{TIME} \cup \{\infty\}$ , and  $l \leq u$ ;
- Formulae:  $\phi ::= a \mid t_1 \supset t_2 \mid \neg \phi \mid \phi \rightarrow \phi' \mid [t]\phi \mid []p$ , where  $a \in A$ ,  $t, t_1, t_2 \in AT$ , and  $p \in SP$ .

### 2.2 RETOOL: the Semantics

The semantics of RETOOL is defined over structures that are based on the notion of *timed transition systems* [10]: given a set  $A$  of attribute symbols and a set  $\Gamma$  of action symbols, a *timed frame*  $\mathcal{F}$  for  $A$  and  $\Gamma$  is a sextuple  $(W, \rightarrow, l, u, I, w_0)$ , where

- $W$  is a set of states;
- For each  $g \in \Gamma$ ,  $\xrightarrow{g} \subseteq W \times W$  is the transition relation for action  $g$ ;
- $l$  maps each  $g \in \Gamma$  to an element  $l(g) \in \text{TIME}$ ;
- $u$  maps each  $g \in \Gamma$  to an element  $u(g) \in \text{TIME} \cup \{\infty\}$  such that  $u(g) \geq l(g)$ ;

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<sup>1</sup>In order to make the presentation clearer, the propositional versions of the logics are used, but the reasoning can easily be extended to a first-order context, with typed variables as attributes and arbitrary assignments as actions.

<sup>2</sup>For all practical purposes, time can be modelled by the set of natural numbers and the corresponding  $\leq$  relation

- $I : A \rightarrow 2^W$  is an interpretation of the attributes, where each  $a \in A$  is assigned the set of worlds where  $a$  is true;
- $w_0$  is the initial state.

Every action  $g \in \Gamma$  has a lower bound  $l(g)$  and an upper bound  $u(g)$ . Intuitively, the lower bound defines the minimum delay that must be observed for the transition to take place (provided all the necessary conditions for the occurrence of such a transition are satisfied). The upper bound defines the maximum delay during which the transition must occur (again, provided all necessary conditions are satisfied). Formally, lower and upper bounds are defined through the use of the notion of *computation*:

A *timed state sequence* [10] for a timed frame is a pair  $\rho = \langle \sigma, T \rangle$ , where  $\sigma$  is an infinite sequence of states ( $\sigma_i \in W$ ) and  $T$  is an infinite sequence of corresponding times ( $T_i \in \text{TIME}$ ), satisfying:

- monotonicity: for all  $i \geq 0$ , either  $T_{i+1} = T_i$ , or  $T_{i+1} > T_i$  and  $\sigma_{i+1} = \sigma_i$ .
- progress: for every  $t \in \text{TIME}$ , there is  $i \geq 0$  such that  $T_i \geq t$ .

A *computation* [10] over a timed frame is a timed state sequence  $\langle \sigma, T \rangle$  such that

- $\sigma$  is a computation of the underlying transition system, i.e., for every  $i \geq 0$ , there is a transition  $\sigma(i)$  such that  $\sigma_i \xrightarrow{\sigma(i)} \sigma_{i+1}$ ;
- (lower bound): for every  $i \geq 0$  in the domain of  $\sigma$ , there is a  $j \leq i$  such that  $T_i - T_j > l(\sigma(i))$  and  $\sigma(i)$  is enabled in every state  $\sigma_k$  for  $j \leq k \leq i$ .
- (upper bound): for every  $g \in \Gamma$  and  $i \geq 0$ , there is  $j \geq i$  with  $T_j - T_i \leq u(g)$  such that either  $g$  is not enabled at  $\sigma_j$  or  $g = \sigma(j)$ .

The denotation of a state proposition  $p$  in a timed frame  $\mathcal{F}$  is the set of states defined as follows:

- $\llbracket a \rrbracket = I(a)$ ;
- $\llbracket \neg p \rrbracket^{\mathcal{F}} = W \setminus \llbracket p \rrbracket^{\mathcal{F}}$ ;
- $\llbracket p \rightarrow p' \rrbracket^{\mathcal{F}} = (W \setminus \llbracket p \rrbracket^{\mathcal{F}}) \cup \llbracket p' \rrbracket^{\mathcal{F}}$ .

The denotation of an action term  $t$  in a timed frame  $\mathcal{F}$  is the set of transitions defined as follows (where  $en(g)$  is the set of states where  $g$  is enabled):

- $\llbracket g \rrbracket^{\mathcal{F}} = \{(w, w') \mid w \xrightarrow{g} w'\}$ ;
- $\llbracket p_i \delta^u q \rrbracket^{\mathcal{F}} = \{(w, w') \mid \exists g \in \Gamma [(w \xrightarrow{g} w') \wedge en(g) \subseteq \llbracket p \rrbracket^{\mathcal{F}} \wedge \forall v, v' ((v \xrightarrow{g} v') \Rightarrow (v' \in \llbracket q \rrbracket^{\mathcal{F}})) \wedge (l \leq l(g) \leq u(g) \leq u)]\}$

Note that the denotation of an action term built with the  $\delta$  operator  $- p_i \delta^u q -$  is the set of all transitions labeled by actions whose necessary condition is  $p$ , whose postcondition is  $q$ , and whose time limits are  $l$  and  $u$ .

Finally, the satisfaction of a formula by a timed frame  $\mathcal{F}$  at a state  $w$  is defined by:

- $\mathcal{F}, w \models p$  iff  $w \in \llbracket p \rrbracket^{\mathcal{F}}$ ;
- $\mathcal{F}, w \models (t_1 \supset t_2)$  iff  $\llbracket t_1 \rrbracket^{\mathcal{F}} \subseteq \llbracket t_2 \rrbracket^{\mathcal{F}}$ ;
- $\mathcal{F}, w \models \neg\phi$  iff not  $\mathcal{F}, w \models \phi$ ;
- $\mathcal{F}, w \models \phi \rightarrow \phi'$  iff  $\mathcal{F}, w \models \phi$  implies  $\mathcal{F}, w \models \phi'$ ;
- $\mathcal{F}, w \models [t]\phi$  iff  $\mathcal{F}, w' \models \phi$  for every  $w'$  such that  $(w, w') \in \llbracket t \rrbracket^{\mathcal{F}}$ ;
- $\mathcal{F}, w \models []p$  iff  $\mathcal{F}, w_0 \models p$ .

“ $\supset$ ” is the *subsumption* operator. To say that action term  $t_1$  subsumes action term  $t_2$  is to say that every action denoted by  $t_1$  is also denoted by  $t_2$  (but not necessarily the other way around). Subsumption can be seen as a refinement on actions: the actions denoted by  $t_1$  refine those denoted by  $t_2$ .

### 2.3 An Axiomatization for RETOOL

The axiom schemes and rules of inference in Figure 2.3 comprise an adequate axiomatization of RETOOL. We assume given an adequate calculus for deriving properties involving members of TIME and the relation  $\leq$ .

In the axiomatization,  $\Lambda$  represents a set of RETOOL formulae, the derivability relation  $\vdash$  is defined in the usual manner, and *enabled*( $t$ ) is an abbreviation for the formula  $\neg[t]\perp$  (which is true in a given state  $w$  iff there is at least one transition leaving  $w$  labeled by an action in the denotation of  $t$ ). The soundness and completeness of this axiomatization is proved in detail in [1].

### 2.4 MTL

MTL (see [5]) is a linear-time temporal logic with time-bounded operators. Its models are computations of timed transition systems. We extend the language of MTL with the action terms of RETOOL taken as propositions (with the intended meaning that an action term  $t$  is true at a given point  $\sigma_i$  of a computation iff the denotation of  $t$  contains the action responsible for the transition from  $\sigma_i$  to  $\sigma_{i+1}$ ). We also add subsumption formulae of the form  $t_1 \supset t_2$  to MTL, yielding the following language over a set  $A$  of attribute symbols, a set  $\Gamma$  of action symbols, and order (TIME,  $\leq$ ) as defined for RETOOL:

$$\tau ::= a \mid t \mid t_1 \supset t_2 \mid \neg\tau \mid \tau \rightarrow \tau' \mid \mathbf{X}_{Rc} \tau \mid \tau_1 \mathbf{U}_{Rc} \tau_2$$

where  $a \in A$ , and  $t, t_1, t_2 \in AT$ ,  $c \in \text{TIME} \cup \{\infty\}$ , and  $R$  is a relation on  $\text{TIME} \cup \{\infty\}$ .

The semantics of this language is defined over a computation  $\langle \sigma, T \rangle$  as follows:

- $\sigma_i, T_i \models a$  iff  $\sigma_i \in I(a)$ ;
- $\sigma_i, T_i \models t$  iff  $\sigma(i) = t$ ;
- $\sigma_i, T_i \models t_1 \supset t_2$  iff  $\llbracket t_1 \rrbracket \subseteq \llbracket t_2 \rrbracket$ ;
- $\sigma_i, T_i \models \neg\tau$  iff not  $\sigma_i, T_i \models \tau$ ;

- (PC) All axioms and rules of propositional calculus
- (K)  $[t](\phi \rightarrow \psi) \rightarrow ([t]\phi \rightarrow [t]\psi)$   
 $[ ](p \rightarrow q) \rightarrow ([ ]p \rightarrow [ ]q)$
- (I)  $[ ]p \rightarrow [t][ ]p$   
 $[t][ ]p \rightarrow (enabled(t) \rightarrow [ ]p)$   
 $[ ]\neg p \leftrightarrow \neg [ ]p$
- (N)  $\frac{\Lambda \vdash \phi}{\Lambda \vdash [t]\phi} \quad \frac{\Lambda \vdash p}{\Lambda \vdash [ ]p}$
- ( $\delta$ )  $\frac{\Lambda \vdash enabled(t) \rightarrow p \quad \Lambda \vdash [t]q \quad \Lambda \vdash l \leq l(t) \leq u(t) \leq u}{\Lambda \vdash t \supset p_l \delta^u q}$
- (S1)  $t \supset t$
- (S2)  $(t_1 \supset t_2) \rightarrow ((t_2 \supset t_3) \rightarrow (t_1 \supset t_3))$
- (S3)  $(t_1 \supset t_2) \rightarrow ([t_2]\phi \rightarrow [t_1]\phi)$
- (NC)  $(t \supset p_l \delta^u q) \rightarrow (enabled(t) \rightarrow p)$
- (Post)  $(t \supset p_l \delta^u q) \rightarrow ([t]q)$
- (Bounds)  $(t \supset p_l \delta^u q) \rightarrow (enabled(t) \rightarrow l \leq l(t) \leq u(t) \leq u)$
- (Global- $\supset$ )  $(t_1 \supset t_2) \leftrightarrow [t](t_1 \supset t_2)$
- ( $\delta$ -Bounds)  $l(p_l \delta^u q) = l \quad u(p_l \delta^u q) = u$

**Figure 1: An axiomatization for RETOOL**

- $\sigma_i, T_i \models \tau \rightarrow \tau'$  iff  $\sigma_i, T_i \models \tau$  implies  $\sigma_i, T_i \models \tau'$ ;
- $\sigma_i, T_i \models \mathbf{X}_{Rc} \tau$  iff  $\sigma_i, T_{i+1} \models \tau$  and  $(T_{i+1} - T_i) Rc$ ;
- $\sigma_i, T_i \models \tau_1 \mathbf{U}_{Rc} \tau_2$  iff, for some  $k \geq i$ , it is the case that  $\sigma_k, T_k \models \tau_2$  and  $(T_k - T_i) Rc$ , and, for every  $j$  such that  $i \leq j < k$ , it is the case that  $\sigma_j, T_j \models \tau_1$ .

The temporal operators are  $\text{neXt}$ , referring to the next state in the computation, and  $\mathbf{U}$ , which states the existence of a future state in which  $\tau_2$  holds and until which  $\tau_1$  holds. Notice that these operators are relativized to the intervals determined by the condition  $Rc$ . Other operators can be defined through abbreviations, such as

- $\mathbf{F}_{Rc} \tau = \top \mathbf{U}_{Rc} \tau$  (sometime in the future);
- $\mathbf{G}_{Rc} \tau = \neg(\mathbf{F}_{Rc} (\neg\tau))$  (always in the future).

An axiomatization of MTL can be found in [5]. The proof rules that relate RETOOL and MTL are as follows:

(R1)

$$\frac{\text{enabled}(t) \rightarrow r \quad t \supset p_0 \delta^\infty q}{t \rightarrow r \wedge \mathbf{X}_{=0} q}$$

Rule (R1) deals only with change, i.e. with the transitions performed by actions. Therefore, it uses the delta operator with time bounds 0 and  $\infty$ . It states that  $t$ , when taken, establishes  $q$ . Notice that the post-condition is established in the next state, and that the transition does not take time.

(R2)

$$\frac{\{h \supset \top_0 \delta^\infty (\neg q) \mid h \in \Gamma - g\} \quad g \supset \top_0 \delta^\infty q}{\mathbf{X}_{=0} q \rightarrow g}$$

Rule (R2) allows us to infer that an action occurs by observing that a given proposition was set to true when only that action can establish it as a post-condition. In a way, this rule “completes” R1.

(R3)

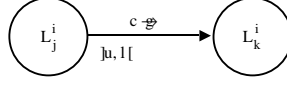
$$\frac{p \rightarrow \neg \text{enabled}(t) \quad t \supset \top_x \delta^\infty \top}{p \rightarrow \mathbf{G}_{\leq x} \neg t}$$

Rule (R3) establishes safety properties by using the lower bound. If  $p$  holds and implies that  $t$  is not enabled, then we know that at least  $x$  units of time have to elapse before  $t$  can be taken, where  $x$  is a lower bound for  $t$ . The temporal operator  $\mathbf{G}_{\leq x}$  means “for the next  $x$  time units”.

(R4)

$$\frac{\{p \rightarrow [h]p \mid h \in \Gamma - g\} \quad p \rightarrow \text{enabled}(g) \quad g \supset \top_0 \delta^y \top}{p \rightarrow \mathbf{F}_{\leq y} g}$$

Rule (R4) establishes liveness properties through the use of the upper bound. If  $p$  holds and is an invariant for all actions other than  $g$ , and  $p$  implies that  $g$  is enabled, then we know that  $g$  will be taken before  $y$  units of time, where  $y$  is an upper bound for  $g$ .



**Figure 2: An edge in a TATD**

(R5)

$$\frac{\begin{array}{ccc} \{\Gamma - g\} & & \{\Gamma - h\} \\ \downarrow & & \downarrow \\ \neg g \vee \neg h & \phi & \phi \end{array}}{\phi}$$

Rule (R5) asserts that if the same conclusion  $\phi$  is obtained assuming that a certain action  $g$  does not happen as well as assuming that a certain other action  $h$  does not happen, and, if those actions never happen together, then we obtain the mentioned conclusion. This is the rule that allows deriving conclusions from non-interfering actions.

### 3 The Concrete Model

In order to allow the software designer to represent a real-time object-oriented system in a manner suitable for formal reasoning, Timed Action Transition Diagrams (TATDs) are introduced as concrete models. This section defines such entities and presents a mapping from TATDs to RETOOL theories.

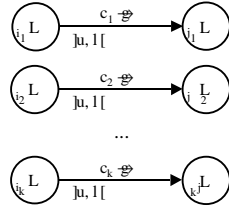
#### 3.1 Timed Action Transition Diagrams

Given a set  $A$  of attribute symbols and a set  $\Gamma$  of action symbols, a Timed Action Transition Diagram (TATD) is a finite directed graph. Each edge in the graph is labelled by a guarded instruction  $c \rightarrow g$ , where  $g \in \Gamma$  and  $c$  is a state proposition, and by a pair  $[l, u]$ , where  $l \in \text{TIME}$  and  $u \in \text{TIME} \cup \{\infty\}$  and  $l \leq u$ .

Each node in the graph represents a location of the flow of control of an object in the system. An edge between two locations  $L_j$  and  $L_k$  of object  $i$  is pictorially represented as in Figure 2.

#### 3.2 Mapping TATDs onto RETOOL Theories

In order for the formal plane to reason about the behavior of the system, the TATD corresponding to each object is mapped onto a RETOOL theory. The integration of the behavior of several objects is achieved by means of colimits in the appropriate category of RETOOL specifications ( $\mathcal{C}_{\text{RETOOL}}$ ), following a well-known approach illustrated by [2] and [8]. For lack of space, we do not provide here further details about the categorical framework for this integration (which takes into account the possibility of various objects of the same class to be active at the same time).



**Figure 3: Edges labelled by  $g$  in a TATD**

Given a TATD over the set  $A_d$  of attribute symbols and the set  $\Gamma$  of action symbols, we produce a set of RETOOL formulae over the following attribute symbols:

$$A = A_d \cup \{atL_0, atL_1, \dots, atL_{n-1}\}$$

where  $L_i, 0 \leq i < n$  are the  $n$  locations in the TATD.

The RETOOL theory for a given TATD consists of the following formulae:

$$atL_i \rightarrow \left( \bigwedge_{i \neq j} \neg atL_j \right)$$

(At each state, the flow of control is at most in one location.)

$$[] \left( \left( \bigwedge_{0 \leq i < n} \neg atL_i \right) \wedge \Theta \right)$$

(The initial state is the one satisfying a set  $\Theta$  of initial conditions. In the initial state, the flow of control is not yet in any location  $L_i$ .)

For each action symbol  $g$  labelling edges in the TATD such as the ones in Figure 3, the following formulae are produced:

$$g \supset \frac{((atL_{i_1} \wedge c_1) \vee (atL_{i_2} \wedge c_2) \vee \dots \vee (atL_{i_k} \wedge c_k))}{\delta^u} (atL_{j_1} \vee atL_{j_2} \vee \dots \vee atL_{j_k})$$

$$enabled(g) \rightarrow ((atL_{i_1} \wedge c_1) \vee (atL_{i_2} \wedge c_2) \vee \dots \vee (atL_{i_k} \wedge c_k))$$

$$atL_{i_1} \wedge c_1 \rightarrow [g]atL_{j_1}$$

$$atL_{i_2} \wedge c_2 \rightarrow [g]atL_{j_2}$$

$$\dots$$

$$atL_{i_k} \wedge c_k \rightarrow [g]atL_{j_k}$$

Finally, the functionality of the actions must be provided through formulae of the form:

$$p_1 \rightarrow [g]q_1$$

$$p_2 \rightarrow [g]q_2$$

$$\dots$$

$$p_n \rightarrow [g]q_n$$



## 4 A Short Example

Consider a machine that can sell cakes and cigars. After it accepts a coin, it is ready to deliver either a cake or a cigar within 10 time units. After the machine delivers the product, it is reset in at most 1 time unit.

The following set of propositional variables will be used for representing the state:

$$OFF, ON, Waiting, DeliveredCake, DeliveredCigar$$

The following actions represent the possible activities of the machine:

$$begin, coin, reset, cake, cigar$$

The behavior of the machine is specified as follows:

1.  $begin \supset OFF \delta^\infty ON$
2.  $coin \supset ON \delta^\infty Waiting$
3.  $cake \supset Waiting \delta^{10} DeliveredCake$
4.  $cigar \supset Waiting \delta^{10} DeliveredCigar$
5.  $reset \supset DeliveredCigar \delta^0 ON$
6.  $reset \supset DeliveredCake \delta^0 ON$
7.  $OFF \leftrightarrow enabled(begin)$
8.  $ON \leftrightarrow enabled(coin)$
9.  $Waiting \leftrightarrow enabled(cake)$
10.  $Waiting \leftrightarrow enabled(cigar)$
11.  $(DeliveredCigar \vee DeliveredCake) \leftrightarrow enabled(reset)$

We omit the axioms that specify that only one propositional variable is true at a time. The proof rules defined in section 2.4 allow us to derive the following properties:

$$ON \rightarrow \neg \mathbf{F}_{\leq 2} (cake) \wedge \neg \mathbf{F}_{\leq 2} (cigar) \quad (1)$$

$$Waiting \rightarrow \mathbf{F}_{\leq 10} (cake \vee cigar) \quad (2)$$

In order to prove (1), we observe that

$$ON \rightarrow \neg Waiting \text{ and } \neg Waiting \rightarrow (\neg enabled(cake) \wedge \neg enabled(cigar))$$

Thus, from

$$cake \supset \top_1 \delta^\infty \top \text{ and } cigar \supset \top_1 \delta^\infty \top$$

we derive

$$ON \rightarrow \mathbf{G}_{\leq 1} \neg cake \text{ and } ON \rightarrow \mathbf{G}_{\leq 1} \neg cigar$$

respectively, using **R3** in both cases. By MTL reasoning, we derive  $ON \rightarrow \mathbf{G}_{\leq 2} \neg cake$  and  $ON \rightarrow \mathbf{G}_{\leq 2} \neg cigar$ . Recall that  $\mathbf{G}_{\leq 2} \neg p = \neg \mathbf{F}_{\leq 2} p$ , by definition.

Let  $\Gamma$  be the set of all actions present in the specification. In order to prove (2), first we observe that

$$\forall g \in ((\Gamma - \{cake\}) - \{cigar\}) : Waiting \rightarrow [g]Waiting \quad (3)$$

$$Waiting \rightarrow enabled(cigar) \quad (4)$$

$$cigar \supset \top_1 \delta^{10} \top \quad (5)$$

Thus, by applying rule **R4** to (3), (4), and (5), we can conclude

$$Waiting \rightarrow \mathbf{F}_{\leq 10} cigar$$

and hence

$$Waiting \rightarrow \mathbf{F}_{\leq 10} (cigar \vee cake)$$

Similarly we have

$$\forall g \in ((\Gamma - \{cigar\}) - \{cake\}) : Waiting \rightarrow [g]Waiting \quad (6)$$

$$Waiting \rightarrow enabled(cake) \quad (7)$$

$$cake \supset \top_1 \delta^{10} \top \quad (8)$$

Thus, by applying rule **R4** to (6), (7), and (8), we can conclude

$$Waiting \rightarrow \mathbf{F}_{\leq 10} cake$$

and hence

$$Waiting \rightarrow \mathbf{F}_{\leq 10} (cigar \vee cake)$$

Thus, by using **R5** and  $\neg cake \vee \neg cigar$ , we reach the desired conclusion.

## 5 Concluding Remarks

The approach adopted for modeling real-time aspects (timed transition systems for the object's lifecycle specification) relies on an extension of Timed Transition Systems and Timed Action Transition Diagrams as presented in [10]. The extension consists in working with a specification level based on the use of action modalities and the  $\delta$  operator. The use of action names allows us to separate methods from their functionality and, therefore, model their reactive and real-time aspects. Action modalities are then used for specifying their functionality (pre/postconditions). The  $\delta$  operator refers to the necessary conditions and time bounds of actions. The Metric Temporal Logic (MTL) of [5] was extended with action terms as propositions and related to their specification by several inference rules.

Space limitations have prevented us from illustrating the proposed approach and explaining how integration of different objects in a system is supported. The framework for integration is based on the categorial approach presented in [2] and [8].

Work in progress includes the search for an automatic theorem-proving strategy for the RETOOL/MTL combination and the application of this framework to different fields, e.g. the design of hypermedia documents, in which real-time constraints also occur naturally.

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